

NSN Oscillator

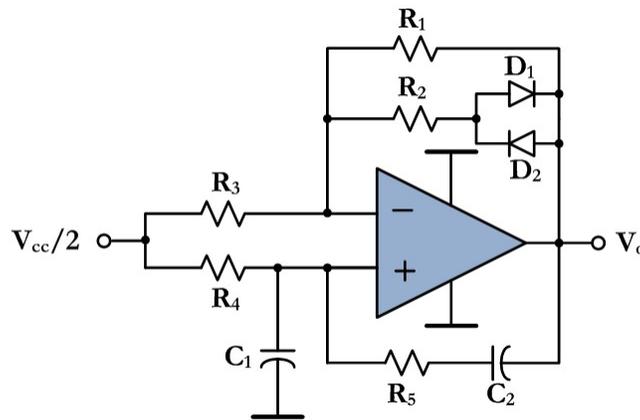
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Circuit Overview

The circuit being analyzed is the following:

Figure 1: Circuit



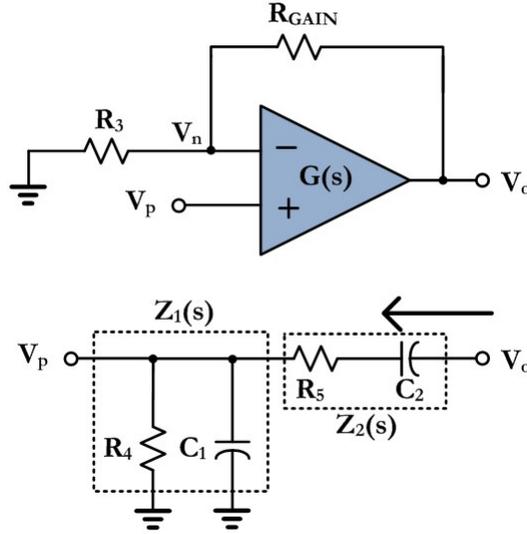
We can see that this circuit has both a positive and negative feedback loop. The positive feedback loop looks like a filter. The negative feedback loop looks like an amplifier. The fact that it has both a positive and negative feedback loop implies that this circuit is likely an oscillator.

Circuit Analysis

AC Analysis

In order to analyze this circuit's AC response, we use AC analysis. In AC analysis, all DC sources are put to ground (including all supplies). To make the circuit analysis easier, we can split the circuit as shown in Figure 2.

Figure 2: Split Circuit



For now, take all components connected between V_o and V_n be lumped into R_{GAIN} .

Positive Feedback Branch

The bottom circuit is the easiest to analyze. As can be seen, it is a voltage divider with the input being V_o and the output being V_p . From the circuit:

$$Z_1(s) = \frac{R_4 \cdot \frac{1}{sC_1}}{R_4 + \frac{1}{sC_1}} = \frac{R_4}{1 + sR_4C_1} \quad (1)$$

$$Z_2(s) = R_5 + \frac{1}{sC_2} \quad (2)$$

The voltage divider transfer function is:

$$\beta(s) = \frac{V_p}{V_o} = \frac{Z_1(s)}{Z_1(s) + Z_2(s)} = \frac{sR_4C_2}{R_4R_5C_1C_2s^2 + (R_4C_1 + R_4C_2 + R_5C_2)s + 1} \quad (3)$$

We can see that the positive feedback circuit is a biquad filter with a zero at DC.

Negative Feedback Branch

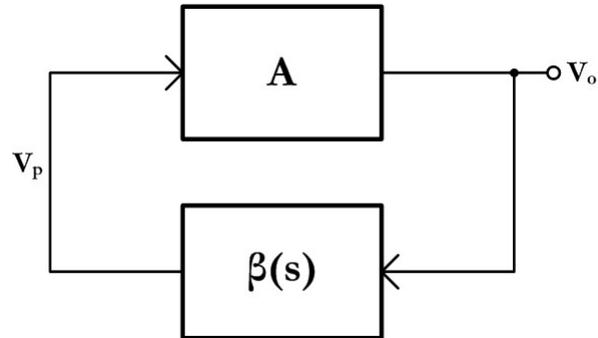
The negative feedback side is a non-inverting amplifier opamp configuration. We will take the gain of the opamp as ideal $G(s) \rightarrow \infty$. Therefore, we can write the top branch as:

$$A(s) = \frac{V_o}{V_p} \approx A = 1 + \frac{R_{GAIN}}{R_3} \quad (4)$$

Loop Gain Analysis

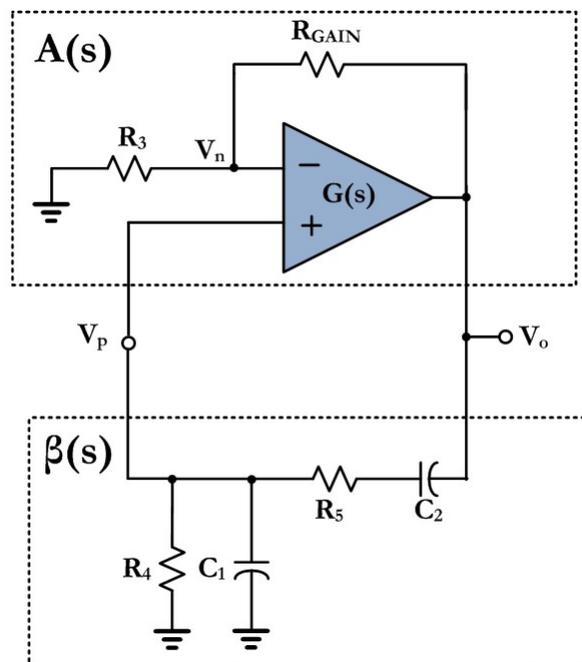
The system block diagram can be summarized as:

Figure 3: Split Circuit



Redrawing using the circuit elements:

Figure 4: Split Circuit



We will begin by using a loop gain approach to analyzing the circuit and determining the conditions required to maintain oscillation.

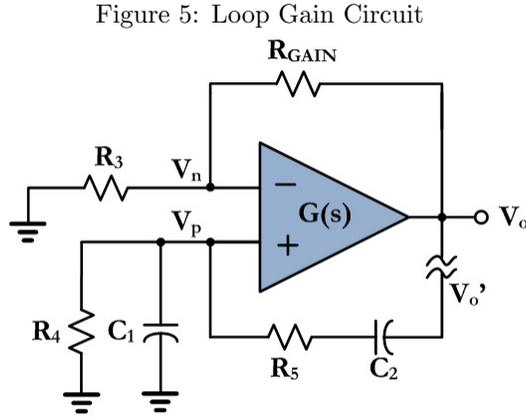
Barkhausen Stability Criterion

According to the Barkhausen Stability Criterion, there are two conditions that must be met in order to maintain stable oscillations. If these are not met, one of two things will happen: the circuit will become

stable and settle to a single voltage (rendering this circuit useless since one can't predict what voltage it will settle to), or the circuit will continue oscillating out of control (unstable oscillation). We desire to control the oscillations to make this circuit a useful circuit. The two conditions to maintain stable oscillations are:

- The loop gain must equal unity: $|\beta \cdot A| = 1$
- There must be positive feedback with a net phase shift around the loop of 0° .

In order to apply the Barkhausen criterion to our circuit, we must first calculate the loop gain. The loop gain is defined as the product of all blocks in the loop. In order to calculate the loop gain, however, the loop must first be broken. We chose to break the loop as follows:



From equations (3) and (4):

$$T(s) = \beta(s) \cdot A(s) = \frac{V_o}{V_p} \cdot \frac{V_p}{V'_o} = \left(1 + \frac{R_{GAIN}}{R_3}\right) \cdot \frac{sR_4C_2}{R_4R_5C_1C_2s^2 + (R_4C_1 + R_4C_2 + R_5C_2)s + 1} \quad (5)$$

This will simplify to:

$$T(s) = \frac{(R_3 + R_{GAIN})R_4C_2s}{R_3 \cdot (R_4R_5C_1C_2s^2 + (R_4C_1 + R_4C_2 + R_5C_2)s + 1)} \quad (6)$$

Recall that $s = j\omega$:

$$T(\omega) = \frac{(R_3 + R_{GAIN})R_4C_2j\omega}{R_3 \cdot (-R_4R_5C_1C_2\omega^2 + (R_4C_1 + R_4C_2 + R_5C_2)j\omega + 1)} \quad (7)$$

We have three poles in this circuit and one zero. In order to guarantee that the phase goes to zero, we need the pole-zero phase cancellation and we also need to cancel the other two poles with the +1. That is:

$$-R_4R_5C_1C_2\omega^2 + 1 = 0 \quad (8)$$

$$\therefore \omega = \frac{1}{\sqrt{R_4R_5C_1C_2}} \quad (9)$$

This will determine your only valid frequency of operation. If we allow $R_4 = R_5 = R$ and $C_1 = C_2 = C$, then the frequency of operation simplifies to:

$$\boxed{\omega = \frac{1}{RC}} \quad (10)$$

Using our simplified equation, keeping our assumption of operation at the resonant frequency, equation (7) simplifies to the following. Notice that the ω^2 term has already been cancelled out with the +1.

$$T(\omega) = \frac{(R_3 + R_{GAIN})R_4C_2j\omega}{R_3 \cdot (R_4C_1 + R_4C_2 + R_5C_2)j\omega} = \frac{(R_3 + R_{GAIN})RC}{R_3 \cdot (RC + RC + RC)} = \frac{R_3 + R_{GAIN}}{R_3 \cdot 3} = 1 \quad (11)$$

Recall that the loop gain must equal 1. We can simplify the loop gain equation until we arrive at a familiar equation: equation (4)!

$$\boxed{1 + \frac{R_{GAIN}}{R_3} = 3 = A(s)} \quad (12)$$

By simplifying, we can see that the gain of the non-inverting amplifier must be equal to 3 in order to comply with the Barkhausen criterion.

Closed Loop Analysis

An alternative approach to finding the criteria for stable oscillations would be to look at the closed loop response. From Figure 3, and knowing that we have positive feedback for $\beta(s)$, we can write Mason's loop equation as:

$$H(s) = \frac{A}{1 - \beta(s) \cdot A} \quad (13)$$

From equations (3) and (4):

$$H(s) = \frac{(R_3 + R_{GAIN}) \cdot (R_4R_5C_1C_2s^2 + (R_4C_1 + R_4C_2)s + 1)}{R_3R_4R_5C_1C_2s^2 + (R_3R_5C_2 + R_3R_4C_1 - R_4R_{GAIN}C_2)s + R_3} \quad (14)$$

We know that in order to get perfectly stable oscillations, all of our poles must lie directly on the $j\omega$ -axis. This implies that there would be purely imaginary poles. We will again use the simplification, $R_4 = R_5 = R$ and $C_1 = C_2 = C$.

$$H(\omega) = \frac{(R_3 + R_{GAIN}) \cdot (-R^2C^2\omega^2 + 2RCs + 1)}{-R_3R^2C^2\omega^2 + (R_3RC + R_3RC - RCR_{GAIN})j\omega + R_3} \quad (15)$$

We can rearrange as follows:

$$H(\omega) = \left(1 + \frac{R_{GAIN}}{R_3}\right) \cdot \frac{(-R^2C^2\omega^2 + 2RCj\omega + 1)}{-R^2C^2\omega^2 + (RC + RC - RCR_{GAIN})j\omega + 1} \quad (16)$$

In order to make the poles purely imaginary, we must cancel out the s term from the characteristic equation (the denominator).

$$2RC - RCR_{GAIN} = 0 \Rightarrow \frac{R_{GAIN}}{R_3} = 2 \quad (17)$$

Going back to equation (4), we get:

$$\boxed{1 + \frac{R_{GAIN}}{R_3} = 3 = A(s)} \quad (18)$$

Now, assuming that we are operating with a gain of 3, we can simplify equation (16) to:

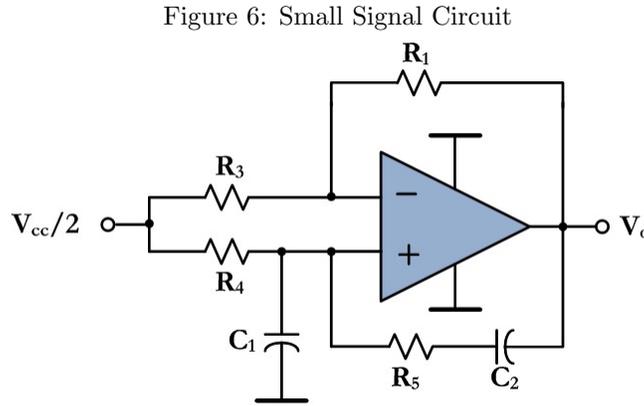
$$H(\omega) = 3 \cdot \frac{(-R^2C^2\omega^2 + 2RCj\omega + 1)}{-R^2C^2\omega^2 + 1} \quad (19)$$

From the characteristic equation, we know that the poles will occur where the denominator of the transfer function goes to zero, therefore, the frequency of oscillation:

$$-R^2C^2\omega^2 + 1 = 0 \Rightarrow \omega = \pm \frac{1}{RC} \quad (20)$$

Automatic Gain Adjustment

As we saw with the Barkhausen criterion and the closed loop analysis, in order to sustain stable oscillations, the gain of the amplifier must be exactly 3. Because opamps aren't ideal, an adjustable gain amplifier is desired in order to regulate the gain to 3. In essence, for very small output swing, the diodes will not conduct and the gain will be:



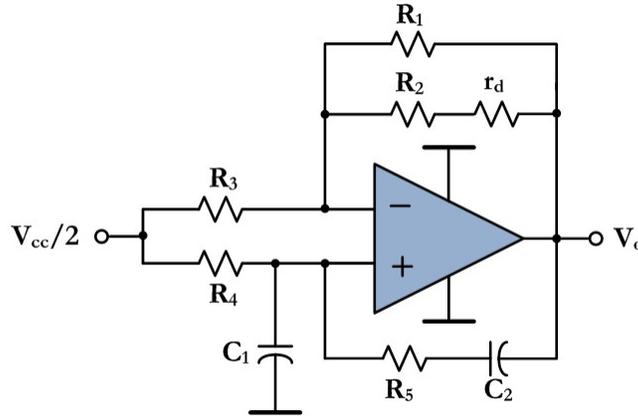
$$A = 1 + \frac{R_1}{R_3} \quad (21)$$

In order to make the circuit make sense:

$$\frac{R_1}{R_3} > 2 \quad (22)$$

For these small signals, we will increase the gain of the amplifier to larger than 3. The oscillator will tend to oscillate out of control. That's when the diodes come in. As soon as the amplitude increases, the diodes will begin conducting again and the gain will drop.

Figure 7: Large Signal Circuit



Now, the gain of circuit will be:

$$A = 1 + \frac{R_1 \parallel (R_2 + r_d)}{R_3} \quad (23)$$

Where r_d is the internal resistance of the diode. These resistors must be picked such that:

$$\frac{R_1 \parallel (R_2 + r_d)}{R_3} \approx 2 \quad (24)$$

MATLAB Code

In order to ease the algebra, the following MATLAB code was written.

```
%Circuito Armando Vera/NSN - Oscilador Senoidal
%Matlab script --- 11-16-2011

clear all;
clc;

%Declaro todas mis variables 'simbolicas'
syms R1 R2 R3 R4 R5 C1 C2 RF s omega;

%Obtengo filtro de retroalimentacion
Z1 = simplify(R4*1/(s*C1) / (R4+1/(s*C1))) %Paralelos C1 y R4
Z2 = R5+1/(s*C2) %Serie R5 y C2
BofS = simplify(Z1/(Z1+Z2)) %Funcion de transferencia del filtro

%Obtengo ganancia A
A = 1+RF/R3

%%%%%%%%%% Usando "Loop Gain" %%%

%Obtengo "Loop Gain"
TofS = simplify(A*BofS)
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```

%Extraigo numerador y denominador de T(s)
[numTofS,denTofS]=numden(TofS)

%Substituyo s=j*omega al denominador
denTof0mega=subs(denTofS,s,i*omega)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Usando "Closed Loop" %%%

%Obtengo "Closed Loop" funcion de transferencia
HofS = simplify(A/(1-A*BofS))

%Extraigo numerador y denominador de H(s)
[numHofS,denHofS]=numden(HofS)

%Substituyo s=j*omega al denominador
denHof0mega=subs(denHofS,s,i*omega)

```

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